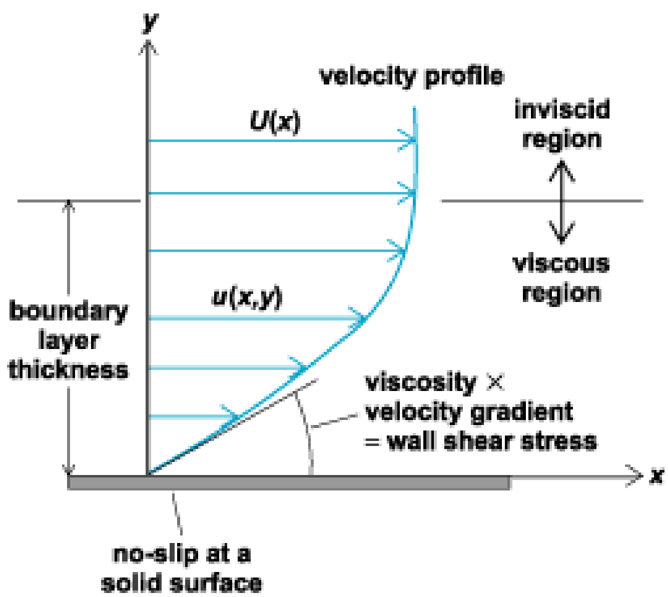
# BOUNDARY LAYER THEORY

## 3.1.1 INTRODUCTION

When a real fluid flows past a solid boundary, a layer of fluid adheres to the boundary surface due to viscosity of the fluid, and a small region is developed in the vicinity of the boundary surface in which the velocity of flowing fluid increases gradually from zero at the boundary surface to the velocity of main stream or free stream velocity. This region is known as *boundary layer*, in which there exists a large velocity gradient ($\partial V/\partial y$) normal to the boundary surface.




3.1 Velocity profile

Within the boundary layer, velocity varies from zero at the boundary surface to free- stream velocity in the direction normal to the boundary. As the velocity gradient exists in this region, fluid exerts a shear stress on the wall in the direction of motion. The velocity of remaining fluid which is outside the boundary layer, is constant and equal to free-stream velocity. as there is no variation of velocity in this region velocity gradient becomes zero and as a result of this, the shear stress is zero.

### 3.1.2 BOUNDARY LAYER THICKNESS

The nominal thickness of the boundary layer δ is considered equal to the distance y from the boundary surface at which the velocity v reaches 99% of the free stream velocity V, i.e., v=0.99V.

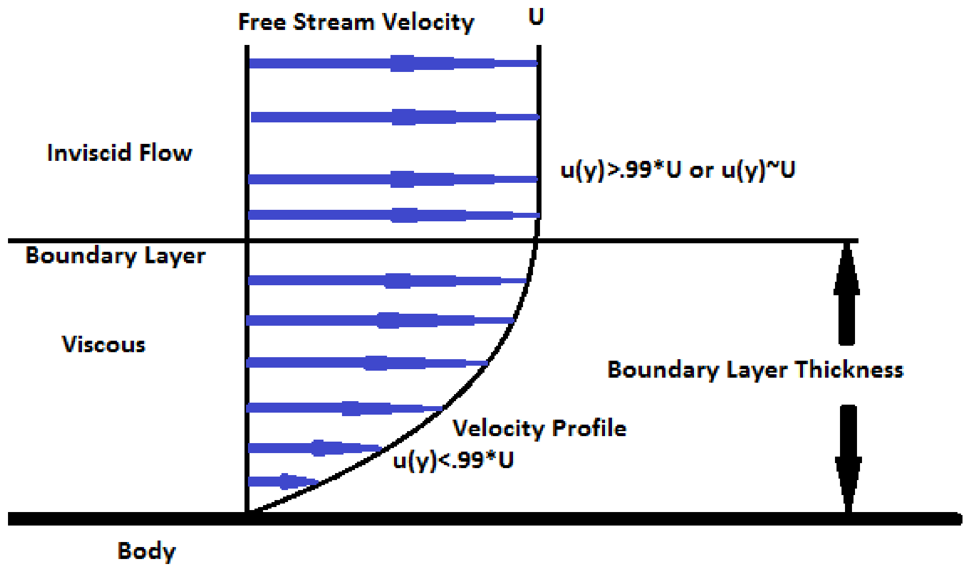



Fig. 3.2 Boundary Layer thickness

The *displacement thickness* $\delta^*$ is defined as the distance by which the boundary surface would have to be displaced outwards so that the total actual discharge would be same as that of an ideal fluid past the displaced boundary. It may be expressed by the equation

$$\delta^* = \int_0^\infty \left(1 - \frac{v}{V}\right) dy$$

The *momentum thickness* θ is defined as the distance from the actual boundary surface such that the momentum flux corresponding to the main stream velocity V though this distance θ is equal to the deficiency or loss of momentum due to the boundary layer formation. It may be mathematically expressed by the equation

$$\theta = \int_0^\infty \frac{v}{V}\left(1 - \frac{v}{V}\right) dy$$

The *energy thickness* $\delta_E$ is defined as the distance from the actual boundary surface such that the energy flux corresponding to the main stream velocity V through this distance is equal to the deficiency or loss of energy due to the boundary layer formation. It may be mathematically expressed by this equation

$$\delta_E = \int_0^\infty \frac{v}{V}\left(1 - \frac{v^2}{V^2}\right) dy$$

### 3.1.3 FORMATION OF BOUNDARY LAYER OVER A FLAT PLATE

Consider the flow of fluid having free stream velocity equal to V, over a thin plate, then the drag force on the plate can be determined by knowing the shear stress at the boundary of the pipe by using Von - Karman momentum integral equation.

Von- Karman momentum integral equation of the boundary layer is given as

$$\frac{\tau_0}{\rho V^2} = \frac{d\theta}{dx}$$

Where $\tau_0$= shear stress at boundary surface

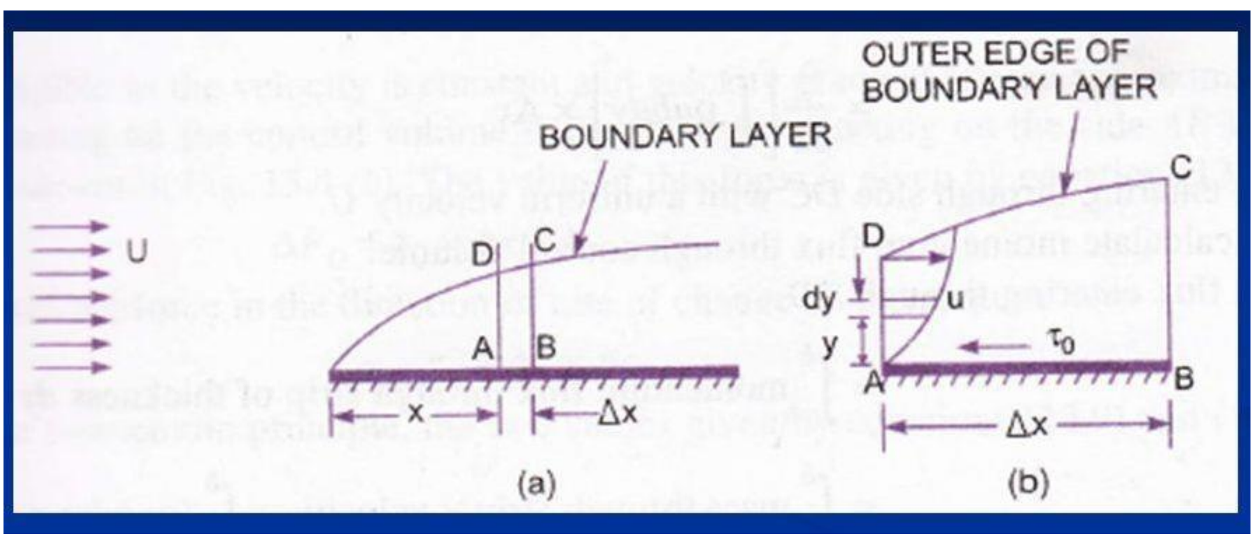



Fig. 3.3 Boundary layer over a flat plate

In a laminar boundary layer the flow exhibits all the characteristics of laminar flow. The velocity distribution in a laminar boundary layer is parabolic. The laminar boundary layer exists when the Reynolds number $Re_x = \frac{Vx}{v}$ is less than $5 \times 10^5$,

Where

V= Free stream velocity of flowing fluid

x= distance from the leading edge of the solid boundary; and

v= kinematic viscosity of the flowing fluid.

The thickness of the laminar boundary layer is given by

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

From the exact analytical solution of the boundary layer equation by Blasius the following expressions for the displacement the thickness and momentum thickness have been obtained

$$\delta^* = \frac{1.729x}{\sqrt{Re_x}}$$

The average drag coefficient $C_f$ obtained by Blasius by the exact solution of the boundary layer equations is given by

$$c_f = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{VL/v}}$$

Where L= length of the plate

In turbulent boundary layer the flow exhibits the characteristics of turbulent flow. The velocity distribution in a turbulent boundary layer follows a logarithmic law i.e., v~log y, which can also be represented by a power law of the type

$$\frac{v}{V} = \left(\frac{y}{\delta}\right)^n$$

The value of the exponent n is approximately (1/7) for moderate Reynolds number ($Re_x < 10^7$) for a flat plate.

Turbulent boundary layer is usually thicker than laminar boundary layer

The thickness of turbulent boundary layer is given by

$$\delta = \frac{0.376x}{(Re_x)^{1/5}}$$

The average drag coefficient $c_f$ for turbulent boundary layer developed along a long thin plate held stationary in a uniform stream in the direction parallel to the flow is given by

$$c_f = \frac{0.074}{(Re_L)^{1/5}}$$

If the plate is covered with both laminar and turbulent boundary layers the drag coefficient $c_f$ is given by

$$c_f = \frac{0.074}{(Re_L)^{1/5}} - \frac{1700}{Re_L}$$

If the plate is smooth, even in the zone of turbulent boundary layer, there exits a very thin layer immediately adjacent to the boundary, in which the flow is laminar. This thin layer is commonly known as *Laminar Sublayer*. The thickness of laminar sublayer is represented by $\delta'$ which is given by

$$\delta' = \frac{11.6v}{V_*}$$

In which $V_* = \sqrt{(\tau_0/\rho)}$ is known as shear or friction velocity.

### 3.1.4 BOUNDARY LAYER SEPARATION

- The boundary layer thickness is considerably affected by the pressure gradient in the direction of flow.
- If $\partial p/\partial x$ is zero, then the boundary layer continues to grow in thickness along a flat plate.
- With the decreasing pressure in the direction of flow i.e. with negative pressure gradient, the boundary layer tends to be reduced in thickness.

- With positive pressure gradient, the boundary layer thickness rapidly decreases the momentum in the boundary layer.
- The adverse (positive) pressure gradient plus the boundary shear decreases the momentum in the boundary layer.
- If kinetic energy of fluid is not able to overcome the adverse pressure gradient and shear, this result in separation of boundary layers.
- The point on the solid at which the boundary layer is on the verge of separation from the surface is called point of separation.
- In turbulent flow momentum transfer is possible i.e, layer near to body surface may take energy from adjacent layers. In laminar flow no momentum transfer is possible hence laminar boundary layer is more prove to early separation than the turbulent boundary layer.

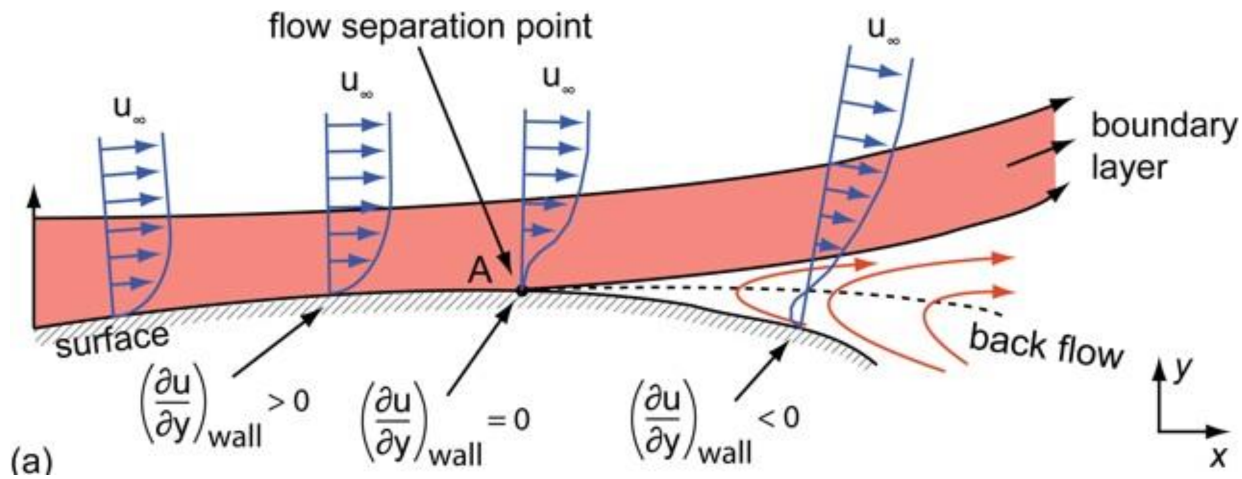



Fig. 3.4 Boundary layer Separation

With the pressure increasing in the direction of flow i.e., with positive pressure gradient, the boundary layer separates from the boundary surface, and back flow and formation of eddies takes place due to which a great loss of energy occur. This phenomenon is called *separation of flow.*

The conditions for attached flow, separation and detached flow are:

(i) For attached flow

$$\left(\frac{\partial V}{\partial y}\right)_{y=0} > 0; \qquad \frac{\partial p}{\partial x} < 0$$

(ii) For separation

$$\left(\frac{\partial V}{\partial y}\right)_{y=0} = 0; \qquad \frac{\partial p}{\partial x} = 0$$

(iii) For detached flow

$$\left(\frac{\partial V}{\partial y}\right)_{y=0} < 0; \qquad \frac{\partial p}{\partial x} > 0$$

Separation of flow occurs in the following cases:

(i) Diffusers

(ii) Pumps

(iii) Fans

(iv) Turbine blades

(v) Aerofoils

#### 3.1.4.1 Effects of boundary layer separation

1. For external flow boundary layer separation leads to increase in pressure drag. Pressure drag is much more than frictional drag.
2. For internal flow, separation leads to increase in flow losses.
3. As separation of boundary layer gives rise to large drag, attempts should be made to void separation by some means.

### 3.1.5 BOUNDARY LAYER CONTROL

The various methods of controlling the formation as well as separation of boundary layer are as follows:

(i) Suction of slow moving fluid by a suction slot

(ii) Supplying additional energy from a blower

(iii) Providing a bypass in the slotted wing

(iv) Rotating boundary in the direction of flow

(v) Providing small divergence in a diffuser

(vi) Providing guide blades inn a bend

(vii) Providing a trip wire ring in the laminar region for the flow over a sphere

# FLOW AROUND SUBMERGED OBJECTS

## 3.2.1 INTRODUCTION

When a fluid is flowing over a stationary body, a force is exerted by fluid on the body. Similarly, when a body is moving in a stationary fluid, a force is exerted by the fluid on the body. Also, when the body and fluid both are moving at different velocities, a force is exerted by fluid on the body. Some of the examples of the fluids over a stationary bodies or bodies moving in a stationary fluid are:

- Flow of air over buildings
- Flow of water over bridges
- Submarines, ships, airplanes and automobiles moving through water or air.
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## 3.2.2 FORCE EXERTED BY A FLOWING FLUID ON STATIONARY BODY

Consider a body held stationary in a real fluid, which is flowing at a uniform velocity U as shown in below figure 3.5.

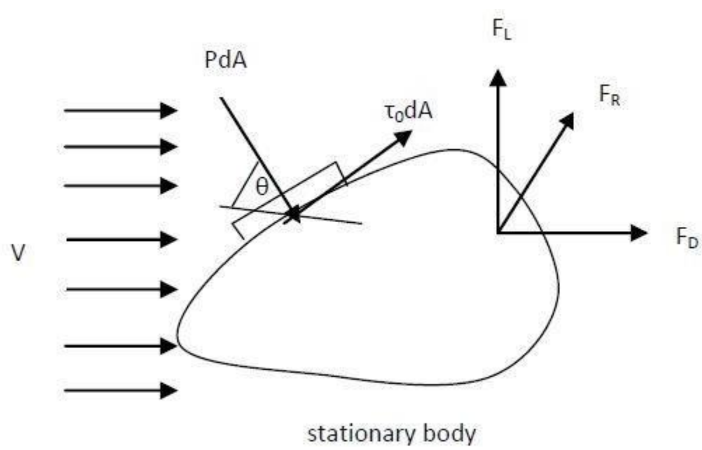



Fig. 3.5 Forces on a submerged body

The fluid will exert a force on a stationary body. The total force ($F_R$) exerted by the fluid on the body is perpendicular to the surface of the body. Thus the total force is inclined to the direction of motion. The total force can be resolved into two components, one in direction of motion and other perpendicular to the direction of the motion.

The component of total force in the direction of motion is called "drag". This component is denoted by $F_D$. It is given by the expression,

$$F_D = \frac{C_D A \rho U^2}{2}$$

Thus drag is the force exerted by the fluid in the direction of motion.

The component of total force in the direction perpendicular to the direction of motion is called "lift". This component is denoted by $F_L$. It is given by the expression,

$$F = \frac{C_L A \rho U^2}{2}$$

Thus drag is the force exerted by the fluid in the direction perpendicular to the direction of motion. Lift force occurs only when the axis of the body is inclined to the direction of fluid flow. If the axis of the body is parallel to the direction of fluid flow, lift force is zero. In that case only drag force acts.

### 3.2.3 TYPES OF DRAG

Total drag on a body is given by the equation,

$$F_D = \int p \cos\theta \, dA + \int \tau_o \sin\theta \, dA$$

where, $\int p \cos\theta \, dA$ = Pressure drag or Form drag

$\int \tau_o \sin\theta \, dA$ = Friction drag or Skin drag or Shear drag

The relative contribution of pressure drag and shear drag to the total drag depends upon:

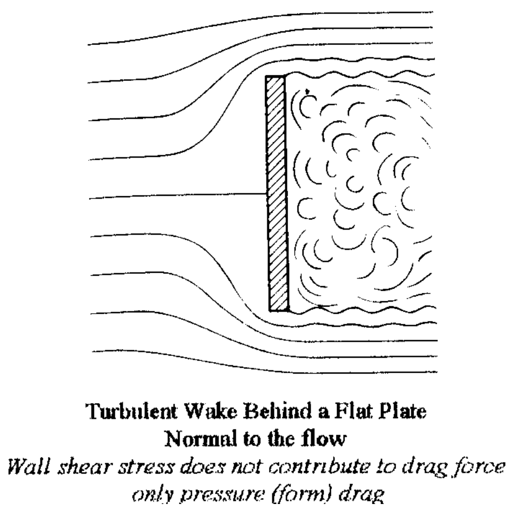
- Shape of the immersed body
- Position of the immersed body in the fluid
- Fluid characteristics

### 3.2.4 DRAG ON A FLAT PLATE

Consider flow of a fluid over a flat plate when the plate is placed parallel to the direction of the flow. In this case, cosθ, which is the angle made by the pressure with the direction of motion will be 90º. Thus, the term ∫ p cosθ dA will be zero and hence the total drag will be equal to friction drag.

If the plate is placed perpendicular to the flow, the angle made by the pressure with the direction of motion will be 0º. Hence the total drag will be due to pressure difference between the upstream and downstream side of the plate.

If the plate is held at an angle with the direction of the flow, then total drag is the sum of the pressure drag and friction drag.



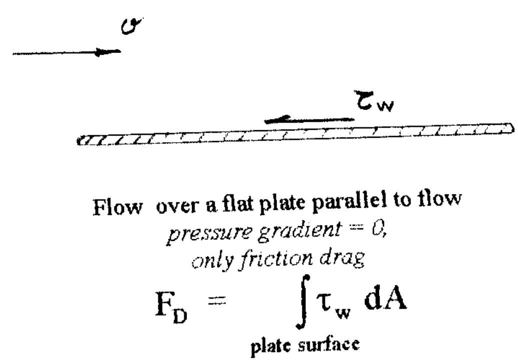




Fig. 3.6 Formation of drag on a flat plate

#### 3.2.4.1 Stream - lined Body

A stream-lined body is defined as that body whose surface coincides with the stream- lines, when the body is placed 'n a flow. In that case the separation of flow will take place only at the trailing edge (or rearmost part of the body). Though the boundary layer will start at the leading edge, will become turbulent from laminar, yet it does not separate up to the rearmost part of the body in the case of stream-lined body. Thus behind a stream-lined body, wake formation zone will be very small and consequently the pressure drag will be very small. Then the total drag on the stream-lined body will be due to friction (shear) only.

A body may be stream-lined

1. At low velocities but may not be so at higher velocities.
2. When placed in a particular position in the flow but may not be so when placed in another position.

#### 3.2.4.2 Bluff Body

A bluff body is defined as that body whose surface does not coincide with the streamlines, when placed in a flow. Then the flow is separated from the surface of the body much ahead of its trailing edge with the result of a very large wake formation zone. Then the drag due to pressure will be very large as compared to the drag due to friction on the body. Thus the bodies of such a shape in which the pressure drag is very large as compared to friction drag are called bluff bodies.

### 3.2.5 DRAG ON A CYLINDER

Consider a real fluid flowing over a circular of diameter D and length Load when the cylinder is placed in the fluid such that its length is perpendicular to the direction of flow. If the Reynolds number of the flow is less than 0.2 ($Re < 0.2$), the inertia force is negligible small as compared to the viscous force and hence the flow pattern about the cylinder will be symmetrical.

As the Reynold number is increased, inertia forces increase and hence they must be taken into consideration for analysis of flow over cylinder. With the increase of the Reynold number, the flow pattern becomes unsymmetrical with respect to an axis perpendicular to the direction of flow. The drag force, i.e., the force exerted by the flowing fluid on the cylinder in the direction of flow depends upon the Reynolds number of the flow.

When a body is placed in a fluid in such a way that its axis is parallel to the direction of fluid flow, the resultant force acting on the body is in the direction of flow. There is no force component on the body perpendicular to the direction of flow. But the component to the force on body perpendicular to the direction of flow, is known as ' Lift'. Hence in this case lift will be zero.

The lift will be acting on the body when the axis of the symmetrical body is inclined to the direction of flow or body is unsymmetrical. In the case of circular cylinder, the body is symmetrical and the axis is parallel to the direction of flow when cylinder is stationary. Hence the lift will be zero. But if the cylinder is rotated, the axis of the cylinder is no longer parallel to the direction of flow and hence lift will be acting on the rotating cylinder. This is explained by considering the following cases:

### 3.2.5.1 Flow of ideal fluid over stationary cylinder

In this case, the flow pattern will be symmetrical and the velocity at any point say C on the surface of the cylinder is given by $u_\theta = 2U\sin\theta$.

where, U = free stream velocity of fluid

$\theta$ = angle made by any point say C on the circumference of the cylinder with the direction of flow.

The velocity distribution over upper half and lower half of the cylinder from the axis of the cylinder are identical and hence the pressure distribution will also be same. hence the lift acting on the cylinder will be zero.

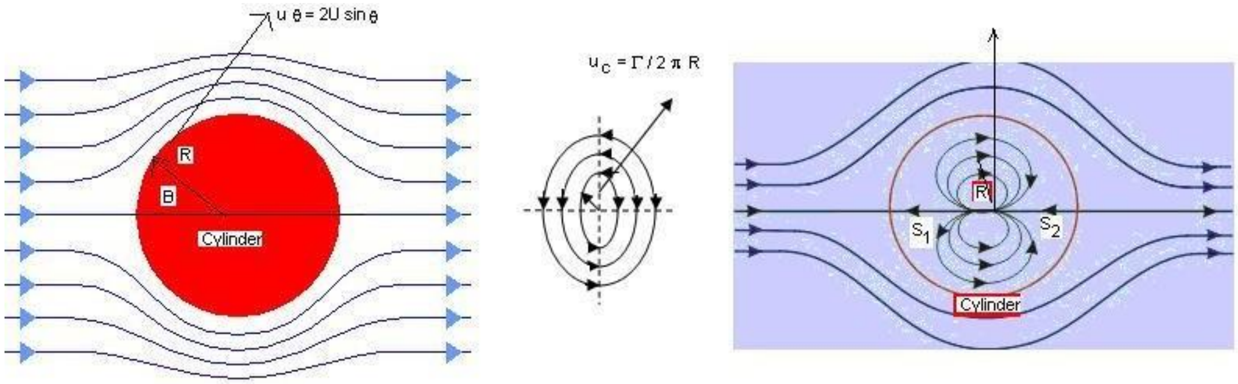
### 3.2.5.2 Flow over cylinder due to constant circulation

The flow along a closed curve is called circulation (i.e., the flow in eddies and vortices). The mathematical concept of circulation is the line integral, taken completely around a closed curve, of the tangential component of the velocity vector.

The flow pattern over a cylinder to which a constant circulation ($\Gamma$) is imparted is obtained by combining the flow patterns and velocity at any point on the surface of the cylinder is obtained by the equation,

$$u = 2U\sin\theta + (\Gamma/2\Pi R)$$

where, R = Radius of the cylinder.




(a) Flow of ideal fluid stationary cylinder

(b) streamline for free vertex

(c) Irrotational flow and circulatory flow superimposed

Fig. 3.7 Formation of drag on a cylinder

For the upper half portion of the cylinder, θ varies from 0° to 180° and hence component of velocity, 2U sinθ is positive. But for the lower half portion of the cylinder, θ varies from 180° to 3600. As sinθ for the values of θ more than 180° and less than 360° is negative and hence component of velocity 2U sinθ will be negative. This means, the velocity on the upper half portion of the cylinder will be more than the velocity on the lower half portion of the cylinder.

But from Bernoulli's theorem we know that at a surface where velocity is less, pressure will be more there and vice-versa. Hence on the lower half portion of cylinder, where velocity is less, pressure will be more than the pressure on the upper half portion of the cylinder. Due to this difference of pressure on the two portions of the cylinder, a force will be acting of the cylinder in a direction perpendicular to the direction of flow. This force is nothing but a lift force. Thus by rotating a cylinder at constant velocity in a uniform flow field, a lift force can be developed.

### 3.2.6 MAGNUS EFFECT

When a cylinder is rotated in a uniform flow, a lift force is produced on the cylinder. This phenomenon of the lift force produced by a rotating cylinder in a uniform flow is known as Magnus Effect.

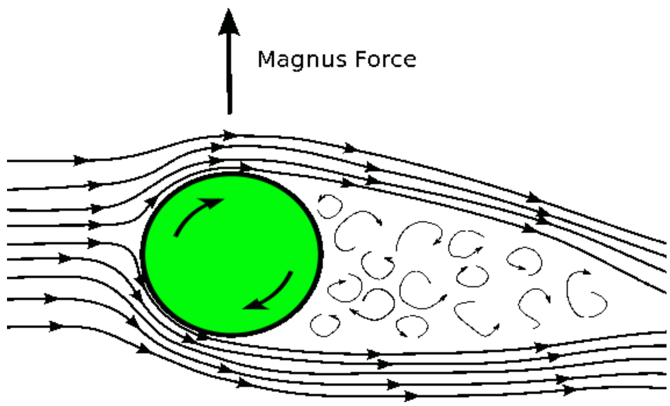



Fig. 3.8 Magnus effect

### 3.2.7 DRAG AND LIFT ON AIRFOILS

An *airfoil* is a two dimensional cross-section of an airplane wing. Alternatively, an airfoil can be thought of as an infinite wing of constant shape.

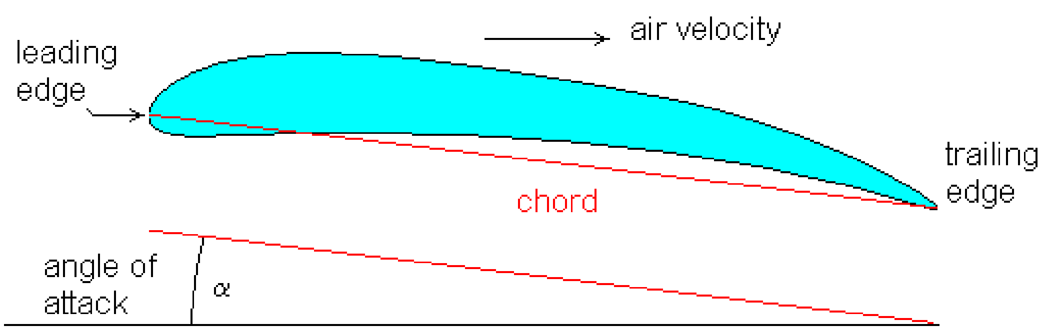



Fig. 3.9 Airfoil

The Magnus effect contains the essential ingredient for the generation of lift by an airfoil--that ingredient is *circulation* of the fluid about the airfoil. Since we are usually interested in large Reynolds number flows, the fluid outside the boundary layer may be treated as being non- viscous. With this assumption, it can be shown that the lift per unit length (or *span*) of the airfoil for a fluid with density and free stream velocity *U*, with a circulation Γ about the airfoil is

$$F_L = \rho UL\Gamma$$

which is known as the *Kutta-Joukowsky Theorem*. This is identical to the result for the cylinder, and the physics is the same. The fluid has a higher velocity on the upper surface of the airfoil than on the lower surface, and the pressure on the upper surface is less than that on the lower surface, leading to lift.

## Numerical problems

1. Air is flowing over a smooth plate with a velocity of 10 m/s. The length of the plate is 1.2 m and width of the plate is 0.8 m. If the laminar boundary layer exists upto a value of $R_e$ =, find the maximum distance from the leading edge upto which laminar boundary layer exists. Find the maximum thickness of laminar boundary layer if the velocity profile is given by –

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2.$$

Take kinematic viscosity for air, ν = 0.15 stokes.

**Sol.** Given, Velocity of air = U = 10 m/s

Length of plate = L = 1.2 m

Width of plate = b = 0.8 m

Reynolds number upto which boundary layer exists is = $2 \times 10^5$

Reynolds number, $R_{ex} = \frac{\rho U x}{\mu} = \frac{\rho U}{\nu}$

If $R_{ex} = 2 \times 10^5$, then *'x'* denotes the distance from the leading edge upto which laminar boundary layer exists.

$$2 x 10^5 = \frac{10\,\boldsymbol{x}}{0.15 x 10^{-4}}$$

$$x = \frac{2 x 10^5 x\, 0.15 x 10^{-4}}{10} = 0.3\, m = 300\, mm$$

The maximum thickness of laminar boundary layer if the velocity profile, $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ is given by equation,

$$\delta = \frac{5.48\, x}{\sqrt{Re_x}} = \frac{5.48\,(0.3)}{\sqrt{2 x 100000}} = 0.00367\, m = \mathbf{3.67\, mm}$$

2. A flat plate 1.5 m x 1.5 m moves at 50 km/hour in stationary air of density 1.15 kg/m$^3$. If the coefficient of drag and lift are 0.15 and 0.75 respectively, determine: i) lift force, ii) drag force, iii) resultant force and iv) power require to keep the plate in motion.

**Sol.** Given, Area of the plate = A = 1.5 m x 1.5 m = 2.25 m$^2$

Velocity of the plate = U = 50 km/hour = $\frac{50 x 1000}{60 x 60} = 13.89\, m/s$

Density of air = 1.25 kg/m$^3$

Coefficient of drag = $C_D$ = 0.15

Coefficient of lift = $C_L$ = 0.75

i) Lift force,

$$F_l = \frac{C_L A \rho U^2}{2} = \frac{0.75 \; x \; 2.25 \; x \; 1.15 \; x \; 13.89^2}{2} = \mathbf{187.2\ N}$$

ii) Drag force

$$F_D = \frac{C_D A \rho U^2}{2} = \frac{0.15 \; x \; 2.25 \; x \; 1.15 \; x \; 13.89^2}{2} = \mathbf{37.44\ N}$$

iii) Resultant force

$$F_R = \sqrt{F_D^2 + F_L^2}$$

$$= \sqrt{37.44^2 + 187.2^2}$$

=**190.85 N**

iv) Power required to keep the plate in motion

P = ( Force in the direction of motion x velocity) / 2

= ($F_D$ x U) / 2

= (37.425 x 13.89) / 2

= 519 W

= **0.519 kW**

3. A man weighing 981 N descends to the ground from an aeroplane with the help of a parachute against the resistance of the air. The shape of the parachute is hemispherical of 2 m diameter. Find the velocity with which the parachute comes down. Assume $C_D$ = 0.5 and density of the air = 1.25 kg/m$^3$.

**Sol.** Given weight of the man = W = 981 N

Then Drag force = $F_D$ = W = 981 N

Diameter of the parachute = D = 2 m

Projected area = A = $\frac{\pi}{4} D^2 = \frac{\pi}{4} 2^2 = \pi$ m$^2$

Coefficient of drag = $C_d$ = 0.5

Density of air = 1.25 kg/m$^3$

Let velocity with which the parachute comes down = U

Total drag is given by the equation,

$$F_D = \frac{C_D A \rho U^2}{2}$$

$$981 = \frac{0.5\ x\ \pi\ x\ 1.25\ x\ U^2}{2}$$

and U = **31.61 m/s**

4. The air is flowing over a cylinder of diameter 50 mm and infinite length with a velocity of0.1 m/s. Find the total drag, shear drag and pressure drag on 1m length of the cylinder is the total drag coefficient is equal to 1.5 and shear drag coefficient is equal to 0.2. Take density of air as 1.25 kg/m$^3$.

**Sol.** Given, Diameter of the cylinder, D = 50 mm = 0.05 m

Length of the cylinder, L = 1 m

Then Projected area = L x D = 1 x 0.5 = 0.05 m$^2$

Velocity of air, U = 0.1 m/s

Total drag coefficient = $C_{DT}$ = 1.5

Density of air = 1.25 kg/m$^3$

Total drag is given by the equation,

$$F_D = \frac{C_D A \rho U^2}{2}$$

$$= \frac{1.5\ x\ 0.05\ x\ 1.25\ x\ 0.1^2}{2} = 0.000468\ N$$

Shear drag is given by the equation,

$$F_{Ds} = \frac{C_{Ds} A \rho U^2}{2}$$

$$= \frac{0.2\ x\ 0.05\ x\ 1.25\ x\ 0.1^2}{2} = 0.0000625\ N$$

Total drag = Pressure drag + Shear drag

Pressure drag = Total drag - Shear drag

$$= 0.000468 - 0.0000625 = \mathbf{0.0004055\ N}$$

**Exercise problems:**

1. Find the displacement thickness, the momentum thickness and energy thickness for the velocity distribution in the boundary layer given by $\frac{u}{U} = \frac{y}{\delta}$, where u is the velocity at a distance y from the plate and u = U at y = δ, where δ = boundary layer thickness. Also calculate the value of $\delta^*/\theta$.

2. Find the displacement thickness, the momentum thickness and energy thickness for the velocity distribution in the boundary layer given by $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$.

3. For the velocity profile for laminar boundary layer flows given as $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$. Find an expression for boundary layer thickness, shear stress and co-efficient of drag in terms of Reynolds number.

4. A thin plate is moving in still atmospheric air at a velocity of 5 m/s. the length of the plate is 0.6 m and width 0.5 m. calculate the thickness of the boundary layer at the end of the plate and drag force on one side of the plate. Take density of air as 1.24 kg/m$^3$ and kinematic viscosity 0.15 stokes.

5. A man weighing 90 kgf descends to the ground from an aero plane with the help of a parachute against the resistance of the air. The velocity with which the parachute, which is hemispherical in shape, come down is 20 m/s. Find the diameter of the parachute. Assume $C_D$ = 0.5 and density of the air = 1.25 kg/m$^3$.

6. A kite 0.8 m x 0.8 m weighing 0.4 kgf assumes an angle of 12° to the horizontal. The string attached to the kite makes an angle of 45° to the horizontal. The pull on the string is 2.5kgf when the wind is flowing with a speed of 30 km/hour. Find the corresponding co-efficient of drag and lift. Density of air is given as 1.25 kg/m$^3$.

7. A cylinder rotates at 150 r.p.m with its axis perpendicular in air stream which is having uniform velocity of 25 m/s. The cylinder is 1.5m in diameter and 10m long. Assuming ideal fluid theory, find the circulation, lift force and position of stagnation points. Take density of air as 1.25 kg/m$^3$.